# Model for the Interaction $p+p \to d+\pi^+$ in the BeV Region*

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The reaction $p+p \to d+\pi^+$ in the BeV region is studied within the one-pion-exchange model with final-state interaction between the neutron and proton to form the deuteron. A simple approximate evaluation of the loop integral gives the final differential cross section in terms of the pion-nucleon coupling constant, the deuteron wave function normalization, the $\pi p$ elastic scattering cross section, and the Ferrari-Selleri form factor. Comparisons with the limited experimental results are made and fairly good agreement is obtained. The large forward scattering cross section near c.m. energy 3.0 BeV is explained in terms of the large backward $\pi^+ p$ elastic scattering cross section at $\pi p$ c.m. energy 1.92 BeV [$I=\frac{3}{2}$, $J=\frac{7}{2}$(?) resonance]. Further implications of the calculation are discussed.

## I. INTRODUCTION

THE one-pion-exchange model[1] has been very successful in the interpretation of pion production in proton-proton collisions in the 1–3-BeV region.[2] Several pion-nucleon isobars have been observed, namely, the 1.23-BeV isobar (the 33 resonance) the 1.52-BeV, and the 1.69-BeV isobars. The 1.92-BeV isobar has not yet been observed, although the threshold energy is 2.46 BeV. Presumably, this isobar will also have been produced at energies much above threshold.

Encouraged by the quantitative success of the one-pion-exchange model we attempt here to extend the calculation to the reaction

$$p+p \to d+\pi^+. \tag{1}$$

The physics involved is rather simple, because process (1) is closely related to

$$p+p \to n+p+\pi^+, \tag{2}$$

with a final-state interaction between neutron and proton to form a deuteron.

Although the ideas involved are elementary in nature, there are several features of the problem which deserve comment at the outset before the detailed calculation is presented. First, we consider the number of dynamical variables in reactions (1) and (2). Reaction (1) has two particles in the final state, and we have just two variables, the total c.m. energy $U$, and the four-momentum transfer squared $u=-(p_1-k_2)^2$. Reaction (2) has three particles in the final state, and we have five independent variables. We choose them to be $U$, $u$, defined as in reaction (1), and $s=-(k_1+p_1)^2$, $k_1^2=(n_1-p_1')^2$, and $d^2=(n_1+p_2)^2$. The various symbols for the four-momenta will be defined in Sec. II (see Figs. 1–3).

When the neutron and proton in the final state combine to form the deuteron, the individual constituent nucleons are off the mass shell. But the variable $d^2=(n_1+p_2)^2$ must be equal to $-M_d^2$, $M_d$ being the deuteron mass. As a result we have now six variables, namely, $U$, $u$, $s$, $k_1^2$, $p_2^2$, and $n_1^2$. Clearly, we are left with a four-dimensional integral over $s$, $k_1^2$, $p_2^2$, and $n_1^2$. In other words our model requires the calculation of a loop integration. It is well known that loop integrals involving strongly interacting particles are extremely difficult to handle. Here, we shall appeal to our knowledge that the deuteron is a very loosely bound system, and consequently the contribution to the loop integral will mainly come from the neighborhood where $p_2^2=n_1^2=-M^2$. With this approximation the neutron-proton deuteron vertex is simply related to the deuteron asymptotic normalization, and the nucleon-pion coupling constant and nucleon-pion elastic scattering amplitude can be easily employed in our calculation. We have to emphasize that there are many fundamental uncertainties in our formulation, and the present calculation should be regarded as a first attempt in a quantitative understanding of the problem. However, the numerical results are quite encouraging.

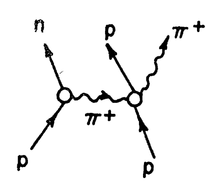



FIG. 1. One-pion-exchange diagram for the process $p+p \to n+p+\pi^+$.

## II. FORMULATION OF THE PROBLEM

First, let us write down the $\pi^+ + p \to \pi^+ + \text{p}$ scattering amplitude, where the incoming pion and the outgoing proton are not on the mass shell. (See Fig. 4.) We define the three variables $s$, $t$, and $u$ as follows:

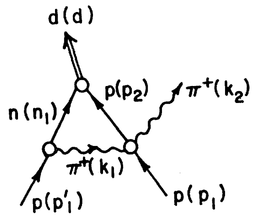
$$\begin{aligned} s&=-(p_1+k_1)^2=-(p_2+k_2)^2,\\ t&=-(p_1-p_2)^2=-(k_2-k_1)^2,\\ u&=-(p_1-k_2)^2=-(p_2-k_1)^2, \end{aligned} \tag{2.1}$$

where $k_1$, $p_1$ are the incoming pion and proton momenta and $k_2$, $p_2$ are the outgoing pion and proton momenta, respectively. We have here

$$p_1^2=-M^2,\quad k_2^2=-\mu^2,\quad p_1+k_1=p_2+k_2$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] C. Goebel, Phys. Rev. Letters **1**, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[2] G. B. Chadwick, G. B. Collins, P. J. Duke, T. Fujii, N. C. Hien, M. A. R. Kemp, and F. Turkot, Phys. Rev. **128**, 1823 (1962). This article also contains a very complete list of references to other works.

FIG. 2. Triangle diagram with $\pi^+$ exchange in $p+p\to d+\pi^+$.




and

$$s+t+u=-[p_1^2+k_1^2+p_2^2+k_2^2]$$
$$=[M^2+\mu^2]-[p_2^2+k_1^2].$$

We choose $k_1^2$, $p_2^2$, $s$, and $u$ as the four independent scalars. The pion-nucleon elastic scattering cross section can now be written down. From Lorentz invariance we have[3]

$$M=A+iB\mathbf{q}+iC\mathbf{r}+D\mathbf{q}\mathbf{r}\,, \tag{2.2}$$

where

$$q=\tfrac{1}{2}(k_1+k_2)\,,\quad r=\tfrac{1}{2}(k_1-k_2)\,.$$

If the outgoing proton is on the mass shell, $p_2^2=-M^2$, then the last two terms in Eq. (2.2) drop out, and we obtain the familiar form. In general, $A$, $B$, $C$, and $D$ are functions of the four invariant scalars, $s$, $u$, $k_1^2$, and $p_2^2$.

The physical amplitude for $\pi^++p$ scattering is obtained by setting $k_1^2=-\mu^2$, $p_2^2=-M^2$ in Eq. (2.2),

$$M=A+iB\mathbf{q}\,, \tag{2.2'}$$

$$\tau=\bar{u}(p_2)[A+iB\mathbf{q}]u(p_1)\,. \tag{2.3}$$

When we sum over the spins of the initial and final protons, we obtain for the square of the matrix element the following form

$$\sum_{\text{spins}}|\tau|^2=\frac{1}{(2M)^2}2\{|A|^2[s+u-2\mu^2+2M^2]+|B|^2[\mu^4-(s-M^2)(u-M^2)]+M(AB^*+A^*B)(u-s)\}\,. \tag{2.4}$$

The cross section for $\pi^++p\to\pi^++p$ is

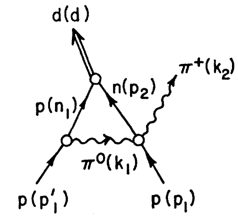
$$\sigma(\pi^++p\to\pi^++p)=\frac{1}{(2\pi)^2}\frac{1}{(2\omega)^2}\left(\frac{M}{E}\right)^2\times\int d^3p_2d^3k_2\frac{\omega E}{k\sqrt{s}}\tfrac{1}{2}\sum_{\text{spins}}|\tau|^2\delta(p_1+k_1-p_2-k_2)$$
$$=\frac{1}{(2\pi)^2}\frac{1}{(2\omega)^2}\left(\frac{M}{E}\right)^2\frac{\omega E}{k\sqrt{s}}\times\int d\Omega_{k_2}k^2\frac{\omega E}{k\sqrt{s}}\tfrac{1}{2}\sum_{\text{spins}}|\tau|^2. \tag{2.5}$$

Therefore,

$$\sum_{\text{spins}}|\tau|^2=(2\pi)^2\left(\frac{2}{M}\right)^2 2s\frac{d\sigma_{\pi p}}{d\Omega_{k_2}}\,, \tag{2.6}$$

[3] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

FIG. 3. Triangle diagram with $\pi^0$ exchange in $p+p\to d+\pi^+$.




where all the quantities are evaluated in the $\pi p$ c.m. system, and

$$E=(k^2+M^2)^{1/2},\quad \omega=(k^2+\mu^2)^{1/2},\quad E+\omega=s^{1/2}.$$

Now we turn our attention to the study of the matrix element for reaction (1). It can be written

$$T=T_1-T_2\,, \tag{2.7}$$

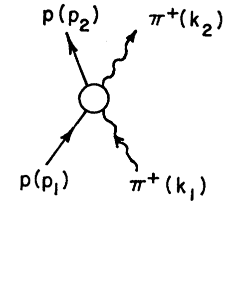
with

$$T_1=\frac{1}{(2\pi)^4}\int\left[\frac{-in_1+M}{n_1^2+M^2-i\epsilon}i\sqrt{2}G\gamma_5u(p_1')\right]^T\bar{\Gamma}\times\frac{-ip_2+M}{p_2^2+M^2-i\epsilon}[A+i\mathbf{q}B+i\mathbf{r}C+\mathbf{q}\mathbf{r}D]\times u(p_1)\frac{1}{k_1^2+\mu^2-i\epsilon}d^4n_1\,. \tag{2.8}$$

The superscript $T$ means taking the transpose of the quantity in bracket. The symbol $G$ denotes the neutron-proton-pion vertex function, and is here a function of two variables, $G=G(n_1^2,k_1^2)$. The symbol $\Gamma$ denotes the neutron-proton-deuteron vertex function, with both the neutron and proton off the mass shell. The scalar functions $A$, $B$, $C$, and $D$ have already been introduced at the beginning of this section. In general, these six functions are extremely complicated and we have no idea whatsoever how they may behave. But as we shall see below, in the approximation we make, these six functions are all related to some experimentally measurable quantities. $T_2$ is obtained from $T_1$ by interchanging $p_1$ and $p_1'$. In Eq. (2.8) we wrote down the contribution from the $\pi^+$ exchange term only. The $\pi^0$ exchange contribution can be similarly written down with appropriate trivial changes. In the following discussion we shall only treat the $\pi^+$ exchange term. In the end when actual numerical calculation is performed we shall, of course, also include the $\pi^0$ exchange contribution.

Since the integrand in Eq. (2.8) contains functions depending on the invariant scalars, $n_1^2$, $p_2^2$, $k_1^2$, and $s$, it

FIG. 4. Diagram for $\pi^++p\to\pi^++p$.



will facilitate the evaluation of the integral if we change the integration variable from $dn_1^4$ to $dn_1^2\,dp_2^2\,dk_1^2\,ds$. The Jacobian of the transformation can be obtained in a straightforward manner.

$$d^4n_1=\frac{1}{16\epsilon^{\alpha\beta\mu\nu}p_{1\alpha}k_{2\beta}k_{1\mu}p_{1\nu}'}dn_1^2dp_2^2dk_1^2ds\,, \quad (2.9)$$

where $\epsilon^{\alpha\beta\mu\nu}$ is the totally antisymmetric tensor.

In the rest system of the deuteron,

$$n_1^2=n_{1d}^2-n_{10d}^2\,, \quad (2.10)$$

$$p_2^2=(d-n_1)^2=-M_d^2+n_1^2+2M_dn_{10d}. \quad (2.11)$$

The integrand in Eq. (2.8) has poles at $n_1^2=-M^2$ and $p_2^2=-M^2$. Therefore, the contribution to the $n_1^2$ and $p_2^2$ integration will come mainly from those neighborhoods. The range of integration can thus be effectively extended from $-\infty$ to $\infty$. Equation (2.8) now becomes

$$T_1=\frac{1}{(2\pi)^4}\int ds\int dk_1^2\int_{-\infty}^{\infty}dn_1^2\int_{-\infty}^{\infty}dp_2^2\frac{1}{16\epsilon^{\alpha\beta\mu\nu}p_{1\alpha}k_{2\beta}k_{1\mu}p_{1\nu}'}$$
$$\times\left[\frac{-in_1+M}{n_1^2+M^2-i\epsilon}i\sqrt{2}G\gamma_5u(p_1')\right]^T$$
$$\times\bar{\Gamma}\frac{-ip_2+M}{p_2^2+M^2-i\epsilon}[A+iqB+irC+qrD]$$
$$\times u(p_1)\frac{1}{k_1^2+\mu^2-i\epsilon}. \quad (2.12)$$

In order to proceed further we make the drastic assumption that the integrand behaves nicely in the upper half-plane and at infinity in both the $n_1^2$ and the $p_2^2$ planes. The infinite line integrals can now be transformed into contour integrals, and the residue at the poles can be extracted.

$$T_1=\frac{1}{(2\pi)^4}(2\pi i)^2\int ds\int dk_1^2\frac{1}{16\epsilon^{\alpha\beta\mu\nu}p_{1\alpha}k_{2\beta}k_{1\mu}p_{1\nu}'}$$
$$\times[(-in_1+M)i\sqrt{2}G\gamma_5u(p_1')]^T\bar{\Gamma}(-ip_2+M)$$
$$\times[A+iqB]u(p_1)\frac{1}{k_1^2+\mu^2-i\epsilon}, \quad (2.13)$$

where all quantities in the integrand are now evaluated at $n_1^2=p_2^2=-M^2$. The $C$ and $D$ terms in the integrand have now vanished, because for $C$

$$\begin{aligned}(-ip_2+M)ru(p_1)&=\tfrac{1}{2}(-ip_2+M)(k_1-k_2)u(p_1)\\&=\tfrac{1}{2}i(-ip_2+M)(-ip_2-M)u(p_1)\\&=-\tfrac{1}{2}i(p_2^2+M^2)u(p_1)=0,\end{aligned}$$

and the $D$ term can now be incorporated into $A$ and $B$. The $s$ integral can easily be transformed into an azimuthal integration. In the deuteron rest system

$$ds=2\frac{1}{M_dp_{1d}'}\epsilon^{\alpha\beta\mu\nu}p_{1\alpha}k_{2\beta}k_{1\mu}p_{1\nu}'d\varphi. \quad (2.14)$$

The ranges of integration for $k_1^2$ can be seen from the relation

$$\begin{aligned}k_1^2&=(p_1'-n_1)^2\\&=-2M^2-2n_1\cdot p_1'\\&=-2M^2-2n_{1d}p_{1d}'\cos\beta+M_dp_{10d}'.\end{aligned} \quad (2.15)$$

In the deuteron rest system,

$$n_{10d}=\tfrac{1}{2}M_d\,,$$
$$n_{1d}=i\kappa\,,$$

and $\kappa^2=M\epsilon$, $\epsilon$ being the binding energy of the deuteron.

Since the binding energy of the deuteron is so low, the $k_1^2$ integration can well be approximated by the product of the average value of the integrand and the width of the integral. The width is $4i\,\kappa\,p_{1d}'$, proportional to the square root of the binding energy.

Equation (2.13) is now reduced to

$$T_1=\frac{1}{(2\pi)^4}(2\pi i)^2\frac{4\pi}{16M_dp_{1d}'}4i\kappa p_{1d}'$$
$$\times[(-in_1+M)i\sqrt{2}G\gamma_5u(p_1')]^T$$
$$\times\bar{\Gamma}(-ip_2+M)[A+iqB]u(p_1)\frac{1}{k_1^2+\mu^2}, \quad (2.16)$$

where all quantities in Eq. (2.16) are to be evaluated at

$$p_2^2=n_1^2=-M^2\,,$$

and

$$k_1^2=-2M^2+M_dp_{10d}'.$$

The function $G$ is now evaluated at $n_1^2=-M^2$, and is therefore simply related to the pionic form factor of the nucleon. The function $\Gamma$ is evaluated at $p_2^2=n_1^2=-M^2$, and is related to the deuteron wave function normalization. Here we shall use the form given by Blankenbecler *et al.*[4]

$$\begin{aligned}\Gamma&=\frac{4\pi N}{M}\left[1+\frac{\rho}{8^{1/2}}S_{12}(\hat{k})\right]\frac{M_d-id}{2\sqrt{2}M_d}(\gamma\cdot\xi)C\\&=\frac{4\pi N}{M}\frac{M_d-id}{2\sqrt{2}M_d}\left[\left(1+\frac{\rho}{8^{1/2}}\right)(\gamma\cdot\xi)-\frac{3\rho}{\sqrt{2}k^2}(k\cdot\xi)(k\cdot\gamma)\right]C,\end{aligned} \quad (2.17)$$

where

$$C=\gamma_0\gamma_2\,,\quad C^2=1\,,\quad C\gamma_\mu{}^TC=-\gamma_\mu\,,\quad\text{and}\quad C\gamma_5C=\gamma_5.$$

$$N^2=\frac{\kappa}{2\pi}\frac{1}{1+\rho^2}\frac{1}{1-\kappa\gamma_e},\quad\text{and}\quad\bar{\Gamma}=\gamma_4\Gamma^+\gamma_4.$$

[4] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (Paris) **2**, 226 (1957), especially pp. 250–252; R. Blankenbecler, M. L. Goldberger, and F. R. Halpern, Nucl. Phys. **12**, 629 (1959).

$\rho$ is a measure of the amount of $D$ state in the deuteron, and $\rho^2\approx 4\%$. $\gamma_e$ is the effective range. $\xi$ is the deuteron polarization vector, and $d\cdot\xi=0$, $d\cdot k=0$, $k^2=-\kappa^2=-M\epsilon$, where $k=\frac{1}{2}(n_1-p_2)$, and $\epsilon=2M-M_d$.

The square of the matrix element $T$ is

$$|T|^2=|T_1-T_2|^2 \approx |T_1|^2+|T_2|^2, \tag{2.18}$$

where we have neglected the interference term. We will see presently that when $T_1$ is large, $T_2$ is small, and vice versa. Therefore, the interference term is probably not too important. When we sum over the final deuteron spin and average over the initial proton spins, we obtain (for details see Appendix)

$$\frac{1}{4}\sum_{\text{spins}}|T_1| =\frac{1}{(2\pi)^2}\left(\frac{\kappa}{2M_d}\right)^2\frac{1}{(k_1^2+\mu_2^2)^2}\frac{1}{4}2G^2k_1^2\left[\frac{4\pi N}{M}\frac{1}{2\sqrt{2}M_d}\right]^2 \times\left\{3\left(1+\frac{\rho}{\sqrt{8}}\right)^2-2\frac{3\rho}{\sqrt{2}}\left(1+\frac{\rho}{8^{1/2}}\right)+\left(\frac{3\rho}{\sqrt{2}}\right)^2\right\} \times(2MM_d+M_d^2)^2\left[(2\pi)^2\left(\frac{2}{M}\right)^2 2s\frac{d\sigma_{\pi p}}{d\Omega_{k_2}}\right]F^2(k_1^2), \tag{2.19}$$

where we have introduced the Ferrari-Selleri off the mass-shell correction function $F(k_1^2)$,[5]

$$F(k_1^2)=\frac{1}{1+(k_1^2+\mu^2)/\alpha},\quad \alpha\sim 60\,\mu^2,$$

and $G$ is now just the pion nucleon coupling constant with $G^2/4\pi=15$.

In the center-of-mass system the differential scattering cross section for reaction (1.1) is

$$\frac{d\sigma}{d\Omega_d}\cong\frac{1}{(2\pi)^2}\left(\frac{2M}{U}\right)^2\frac{d_UM_d}{8p}\frac{1}{4}\sum_{\text{spins}}[|T_1|^2+|T_2|^2], \tag{2.20}$$

where

$$(d_U^2+M_d^2)^{1/2}+(d_U^2+\mu^2)^{1/2}=U,$$
$$d_{0U}=(d_U^2+M_d^2)^{1/2},$$
$$2(p^2+M^2)^{1/2}=U.$$

So far we have neglected the $\pi^0$ exchange contribution, which can be easily included. Instead of $\sqrt{2}\,G$ in Eq. (2.16) we have $G$, and instead of $A$ and $B$ (which should be properly written as $A_{3/2}$ and $B_{3/2}$) we have $(\sqrt{2}/3)(-A_{3/2}+A_{1/2})$ and $(\sqrt{2}/3)(-B_{3/2}+B_{1/2})$, and instead of $\Gamma$ we have $-\Gamma$. The subscripts $\frac{3}{2}$ and $\frac{1}{2}$ denote the different isotopic contributions. Including the $\pi^0$ contribution $T_1$ now becomes

$$T_1=\frac{1}{4\pi}\frac{\kappa}{M_d}[(-in_1+M)\sqrt{2}G\gamma_5u(p_1')]^T\Gamma(-ip_2+M) \times[(\tfrac{4}{3}A_{3/2}-\tfrac{1}{3}A_{1/2})+i\boldsymbol{q}(\tfrac{4}{3}B_{3/2}-\tfrac{1}{3}B_{1/2})] \times u(p_1)\frac{1}{k_1^2+\mu^2}. \tag{2.21}$$

Correspondingly in Eq. (2.19) $d\sigma_{\pi p}/d\Omega_{k_2}$ should be replaced by

$$\frac{1}{2}\left[3\frac{d\sigma}{d\Omega_{k_2}}(\pi^++p\to\pi^++p)-\frac{d\sigma}{d\Omega_{k_2}}(\pi^-+p\to\pi^-+p) +3\frac{d\sigma}{d\Omega_{k_2}}(\pi^-+p\to\pi^0+n)\right].$$

The final expression for the differential cross section is

$$\frac{d\sigma}{d\Omega_d}=2\frac{3-(3/\sqrt{2})\rho+(27/8)\rho^2}{1+\rho^2}\frac{G^2}{4\pi}F^2(k_1^2)\epsilon\kappa\frac{d_U}{U^2p}\frac{k_1^2}{(k_1^2+\mu^2)^2} \times s\left[3\frac{d\sigma^+}{d\Omega_{k_2}}-\frac{d\sigma^-}{d\Omega_{k_2}}+3\frac{d\sigma^0}{d\Omega_{k_2}}\right]+(p_1\rightleftarrows p_1'), \tag{2.22}$$

where

$$k_1^2=-2M^2+\tfrac{1}{4}[U^2+M_d^2-\mu^2]-d_Up\cos\theta, \tag{2.23}$$

$\theta$ = angle between outgoing deuteron and incoming proton in c.m., $0\leq\theta<\pi/2$ and

$$s=\tfrac{1}{2}[U^2-M_d^2+\mu^2]+M^2.$$

$d\sigma/d\Omega_{k_2}$, the differential $\pi$-$p$ scattering cross section, is to be evaluated at its c.m. energy $s^{1/2}$ and its c.m. scattering angle $\delta$, where we have the relationship

$$u=-(p_1-k_2)^2=M^2+\mu^2-Uk_{20U}+2pd_U\cos\theta =M^2+\mu^2-2p_{10s}k_{20s}-2p_{1s}k_{2s}\cos\delta, \tag{2.24}$$

with

$$k_{20U}=(1/2U)[U^2+\mu^2-M_d^2],$$
$$p_{10s}=(1/2s^{1/2})[s+M^2+k_1^2],$$
$$k_{20s}=(1/2s^{1/2})[s+\mu^2-M^2],$$
$$k_{2s}=[k_{20s}^2-\mu^2]^{1/2}.$$
$$p_{1s}=[p_{10s}^2-M^2]^{1/2}.$$

The second term in Eq. (2.22) is obtained from the first term by interchanging $p_1$ and $p_1'$. This interchange amounts to changing $\cos\theta$ to $-\cos\theta$ in Eqs. (2.23) and (2.24). For $\cos\theta=1$, $k_1^2$ is a minimum, $\cos\delta=-1$, and we see that backward $\pi p$ scattering is very important for deuteron formation in the forward direction (either

[5] E. Ferrari and F. Selleri, Phys. Rev. Letters 7, 387 (1961).

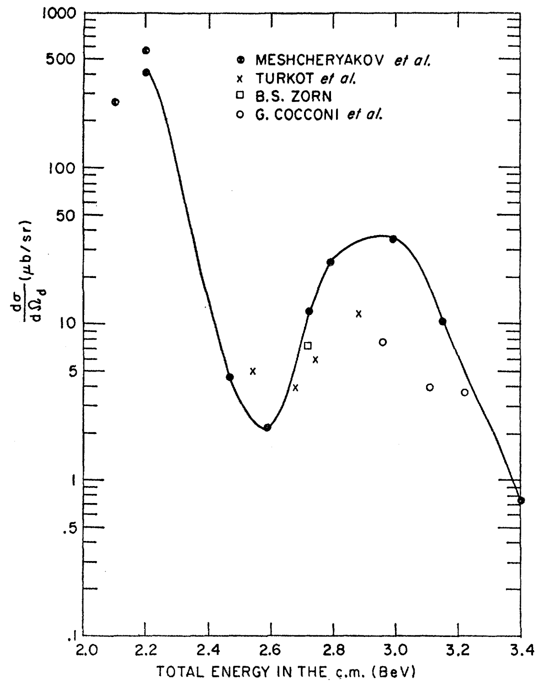



FIG. 5. Differential cross section for $p+p\rightarrow d+\pi^+$ in c.m. system with $\cos\theta=1$. (The solid curve is the calculated curve including backward $\pi p$ scattering only.)

0° or 180° in the c.m. due to the symmetry of the two incident protons).

When $\cos\theta=-1$, $k_1^2$ is very large, and the exchanged virtual pion is very far from the mass shell. Our approximation now becomes highly dubious, since for large momentum transfer ($k_1^2>60\,\mu^2$), we have no reason to expect the one-pion-exchange mechanism to be important. Nevertheless, we have also calculated the contribution from large $k_1^2$. Again we include the Ferrari-Selleri form factor, which is no doubt invalid for such large momentum transfers. It is satisfying that in the peak region for deuteron production we find that the contribution from large $k_1^2$ is much less than that from small $k_1^2$, and consequently does not influence the calculated cross sections in any essential way.

### III. RESULTS AND DISCUSSION

In Eq. (2.18) we have neglected the interference term, which is justified only when one term is much larger than the other. This approximation means that our calculation becomes poorer when we are away from the forward direction, and becomes highly unreliable at 90° in the c.m. system where the interference is largest. Therefore, we have computed the differential cross sections for deuteron production at 0°, $\cos\theta=1$,[6] where we have some available data to compare with.[7,8] (The experiment of Cocconi *et al.* was performed at 60 mrad in the laboratory system. Here, we have made no distinction about these two slightly different directions.) The calculated cross sections are limited in accuracy by our lack of information on the backward scattering cross section in $\pi^-+p\rightarrow\pi^0+n$. From Fig. 5 we see that the calculated cross sections are compatible with the experimental results within a factor of two or three. The outstanding feature of the calculated curve is the prominent peak at $U\sim 3.0$ BeV. Our model explains it in terms of the large backward $\pi^+p$ scattering cross section due to the $I=\frac{3}{2}$, $J=\frac{7}{2}$ resonance at $s^{1/2}=1920$ MeV. This explanation is analogous to the phenomenological theory given by Mandelstam for the other even larger deuteron peak at $U=2.2$ BeV,[9] where the $I=\frac{3}{2}$, $J=\frac{3}{2}$ $\pi p$ resonance plays a dominent role. In fact, in the neighborhood of $U=2.2$ BeV, we obtain a calculated value at 0° which is very close to the experimental value.

The general qualitative agreement between the calculated results and experiments seems to indicate that the one-pion-exchange mechanism may indeed play an important role in deuteron production. However, quantitatively the calculated results are much larger than the experimental cross sections in the peak region near 3.0 BeV. There are several possible sources of error to account for this discrepancy. One major uncertainty is the way the loop integral was evaluated. Another probable source of error may be in the off the mass-shell correction we employed, since it is in the large momentum transfer region (large $k_1^2$) that we have serious discrepancies. Further theoretical studies and experimental investigations are certainly desirable.

Finally we comment briefly on the one-neutron-exchange graph, Fig. 6. It is certainly a simpler graph than the one-pion-exchange graph. But because of this simple structure, we expect its contribution to vary smoothly with energy. In addition, we have the further difficulty of really not knowing how to compute this graph.[10] Since the exchanged neutron is often far away from the mass shell, both vertices are modified in an unknown manner from their on-the-mass shell value. We tend to think that the neutron exchange graph does not play any important role in the energy region of interest here.

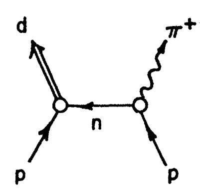



FIG. 6. The neutron exchange graph in $p+p\rightarrow d+\pi^+$.

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[10] See, however, a similar calculation by J. Bernstein, Phys. Rev. **129**, 2323 (1963).

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## APPENDIX

Here we evaluate the square of the matrix element $T_1$. From Eq. (2.16), we have

$$T_1=\frac{1}{2\pi}\frac{\kappa}{\sqrt{2}M_d}G[(-in_1+M)\gamma_5 u(p_1')]^T\Gamma(-ip_2+M)(A+iqB)u(p_1)\frac{1}{k_1^2+u^2}, \tag{A1}$$

and

$$|T_1|^2=\left[\frac{1}{2\pi}\frac{\kappa}{\sqrt{2}M_d}G\frac{1}{k_1^2+\mu^2}\right]^2|M|^2.$$

When we sum over the final deuteron spins and average over the initial proton spins, we get

$$\tfrac{1}{4}\sum_{\text{spins}}|M|^2=\tfrac{1}{4}\sum_{\text{spins}}\text{Tr}\{[(-in_1+M)\gamma_5 u(p_1')\bar{u}(p_1')\gamma_5(-in_1+M)]^T \bar{\Gamma}(-ip_2+M)(A+iqB)\times u(p_1)\bar{u}(p_1)(A^*+iqB^*)(-ip_2+M)\Gamma\}. \tag{A2}$$

Inserting Eq. (2.17) into (A2) and sum over the initial proton spins, (A2) becomes

$$\begin{aligned}\tfrac{1}{4}\sum_{\text{spins}}|M|^2&=\left(\frac{1}{2M}\right)^2\left[\frac{4\pi N}{M}\frac{1}{2\sqrt{2}M_d}\right]^2\frac{1}{4}\sum_{\substack{\text{deuteron}\\ \text{spins}}}'\text{Tr}\Big\{(M_d-id)\Big[\Big(1+\frac{\rho}{\sqrt{8}}\Big)(\gamma\cdot\xi)-\frac{3\rho}{\sqrt{2}k^2}(k\cdot\xi)(k\cdot\gamma)\Big]\\
&\quad\times(in_1+M)(-ip_1'+M)(in_1+M)\Big[\Big(1+\frac{\rho}{\sqrt{8}}\Big)(\gamma\cdot\xi^*)-\frac{3\rho}{\sqrt{2}k^2}(k\cdot\xi^*)(k\cdot\gamma)\Big](M_d-id)\\
&\quad\times(-ip_2+M)(A+iqB)(-ip_1+M)(A^*+iqB^*)(-ip_2+M)\Big\}\\
&=-\left(\frac{1}{2M}\right)^2\left[\frac{4\pi N}{M}\frac{1}{2\sqrt{2}M_d}\right]^2\frac{1}{4}k_1^2\left\{3\Big(1+\frac{\rho}{\sqrt{8}}\Big)^2-2\frac{3\rho}{\sqrt{2}}\Big(1+\frac{\rho}{\sqrt{8}}\Big)+\Big(\frac{3\rho}{\sqrt{2}}\Big)^2\right\}(2MM_d+M_d^2)\\
&\quad\times\text{Tr}\{(M_d-id)(-ip_2+M)(A+iqB)(-ip_1+M)(A^*+iqB^*)(-ip_2+M)\},\end{aligned} \tag{A3}$$

where we have used the relationships,

$$d\cdot\xi=0,\quad d\cdot k=0,$$
$$k^2=k^{*2},\quad k\cdot k^*=-k^2,$$

and

$$\sum_{\substack{\text{deuteron}\\ \text{spin}}}'(\xi^*\cdot\xi)=3, \tag{A4}$$

$$\sum{}'(a\cdot\xi^*)(b\cdot\xi)=(a\cdot b)+\frac{(a\cdot d)(b\cdot d)}{M_d^2}, \tag{A5}$$

$$\sum{}'(k^*\cdot\xi^*)(k\cdot\xi)=-k^2+\frac{(k\cdot d)(k^*\cdot d)}{M_d^2}=-k^2. \tag{A6}$$

The trace calculation can now be done immediately, and the result is

$$\begin{aligned}\tfrac{1}{4}\sum_{\text{spins}}|M|^2&=\frac{1}{4}\frac{1}{(2M)^2}\left[\frac{4\pi N}{M}\frac{1}{2\sqrt{2}M_d}\right]^2k_1^2\left\{3\Big(1+\frac{\rho}{\sqrt{8}}\Big)^2-2\frac{3\rho}{\sqrt{2}}\Big(1+\frac{\rho}{\sqrt{8}}\Big)+\Big(\frac{3\rho}{\sqrt{2}}\Big)^2\right\}2(2MM_d+M_d^2)^2\\
&\quad\times\{|A|^2[s+u-2\mu^2+2M^2]+|B|^2[\mu^4-(s-M^2)(u-M^2)]+M(u-s)(AB^*+A^*B)\},\end{aligned} \tag{A7}$$

where in the last curly bracket we have set $k_1^2=-\mu^2$. With this usual on the mass shell approximation we can now use Eqs. (2.4) and (2.6) to reduce the original rather complicated expression into a simple form. The final result is given by Eq. (2.19).